Exercise 37

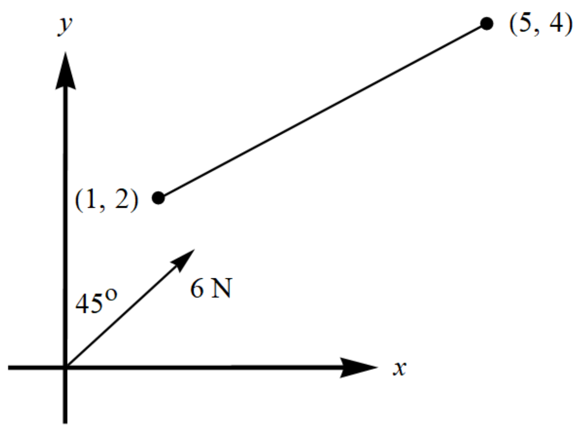
A force of 6 N makes an angle of $\pi/4$ radian with the y axis, pointing to the right. The force acts against the movement of an object along the straight line connecting $(1, 2)$ to $(5, 4)$.

(a) Find a formula for the force vector $\mathbf{F}$.

(b) Find the angle θ between the displacement direction $\mathbf{D} = (5 - 1)\mathbf{i} + (4 - 2)\mathbf{j}$ and the force direction $\mathbf{F}$.

(c) The work done is $\mathbf{F} \cdot \mathbf{D}$, or, equivalently, $\|\mathbf{F}\| \cdot \|\mathbf{D}\| \cos\theta$. Compute the work from both formulas and compare.

Solution



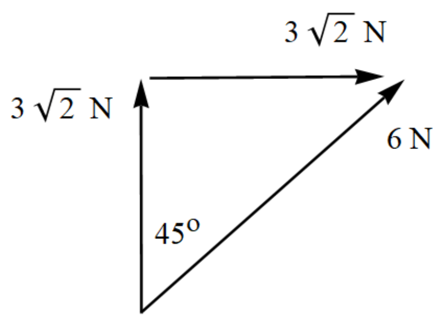
Part (a)

From the figure,

$$\cos 45^\circ = \frac{y}{6} \quad \rightarrow \quad y = 6\cos 45^\circ = 3\sqrt{2}\ \text{N}$$

$$\sin 45^\circ = \frac{x}{6} \quad \rightarrow \quad x = 6\sin 45^\circ = 3\sqrt{2}\ \text{N},$$

so the 6 N force is decomposed as shown below.



Since both components of the force point in the positive x- and y-directions, no minus signs are needed.

$$\mathbf{F} = (3\sqrt{2}, 3\sqrt{2})\ \text{N} = 3\sqrt{2}(1, 1)\ \text{N}$$

Part (b)

Take the dot product of $\mathbf{F} = (3\sqrt{2}, 3\sqrt{2})$ and $\mathbf{D} = (4, 2)$. Let θ be the angle between them.

$$\mathbf{F} \cdot \mathbf{D} = \|\mathbf{F}\|\|\mathbf{D}\| \cos\theta$$

Solve for $\cos\theta$.

$$\begin{aligned}
\cos\theta &= \frac{\mathbf{F} \cdot \mathbf{D}}{\|\mathbf{F}\|\|\mathbf{D}\|} \\
&= \frac{(3\sqrt{2}, 3\sqrt{2}) \cdot (4, 2)}{\sqrt{(3\sqrt{2})^2 + (3\sqrt{2})^2}\sqrt{4^2 + 2^2}} \\
&= \frac{12\sqrt{2} + 6\sqrt{2}}{\sqrt{36}\sqrt{20}} \\
&= \frac{3}{\sqrt{10}}
\end{aligned}$$

Therefore, the angle between the force and displacement vectors is

$$\theta = \cos^{-1}\left(\frac{3}{\sqrt{10}}\right) \approx 18.4^\circ.$$

Part (c)

The work done by the force $\mathbf{F}$ in moving the object from $(1, 2)$ to $(5, 4)$ is

$$\begin{aligned}
W = \mathbf{F} \cdot \mathbf{D} = (3\sqrt{2}, 3\sqrt{2}) \cdot (4, 2) = 12\sqrt{2} + 6\sqrt{2} &= 18\sqrt{2} \text{ N} \cdot \text{(unit of distance)} \\
&\approx 25.5 \text{ N} \cdot \text{(unit of distance)}.
\end{aligned}$$